

Modified Enlarged 18pt

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Friday 12 June 2020 – Afternoon

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Time allowed: 2 hours

plus your additional time allowance

YOU MUST HAVE:

the Printed Answer Booklet sent with the standard paper or any suitable paper provided by the centre. The Printed Answer Booklet may be enlarged by the centre. a scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF



INSTRUCTIONS

Use black ink. You can use an HB pencil, but only for graphs and diagrams.

If you use the Printed Answer Booklet write your answer to each question in the space provided. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.

Fill in the boxes on the front of the Printed Answer Booklet.

Answer ALL the questions.

Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.

The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.

Do NOT send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

The total mark for this paper is 100.

The marks for each question are shown in brackets [].

ADVICE

Read each question carefully before you start your answer.

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FORMULAE

A Level Mathematics A (H240)

Arithmetic series

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n - 1)d\}$$

Geometric series

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \quad \text{for } |r| < 1$$

Binomial series

$$(a + b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n$$

$(n \in \mathbb{N})$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots$$

$$(|x| < 1, n \in \mathbb{R}),$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule $y = \frac{u}{v}, \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x) (f(x))^n dx = \frac{1}{n+1} (f(x))^{n+1} + c$$

Integration by parts $\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$

Small angle approximations

$\sin \theta \approx \theta$, $\cos \theta \approx 1 - \frac{1}{2}\theta^2$, $\tan \theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi \right)$$

Numerical methods

Trapezium rule:

$$\int_a^b y \, dx \approx \frac{1}{2}h \left\{ (y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1}) \right\},$$

$$\text{where } h = \frac{b - a}{n}$$

The Newton-Raphson iteration for solving

$$f(x) = 0: x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B | A) = P(B)P(A | B)$$

$$\text{OR } P(A | B) = \frac{P(A \cap B)}{P(B)}$$

Standard deviation

$$\sqrt{\frac{\sum (x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{OR} \quad \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$,

mean of X is np , variance of X is $np(1 - p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p , the table gives the value of z such that $P(Z \leq z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995
z	0.674	1.282	1.645	1.960	2.326	2.576

p	0.9975	0.999	0.9995
z	2.807	3.090	3.291

Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

SECTION A: Pure Mathematics

Answer ALL the questions.

- 1 Triangle ABC has $AB = 8.5$ cm, $BC = 6.2$ cm and angle $B = 35^\circ$.

Calculate the area of the triangle. [2]

- 2 A sequence of transformations maps the curve $y = e^x$ to the curve $y = e^{2x+3}$.

Give details of these transformations. [3]

- 3 The functions f and g are defined for all real values of x by

$$f(x) = 2x^2 + 6x \text{ and } g(x) = 3x + 2.$$

(a) Find the range of f . [3]

(b) Give a reason why f has no inverse. [1]

(c) Given that $fg(-2) = g^{-1}(a)$, where a is a constant, determine the value of a . [4]

(d) Determine the set of values of x for which $f(x) > g(x)$. Give your answer in set notation. [3]

4 A curve has equation $y = 2\ln(k - 3x) + x^2 - 3x$, where k is a positive constant.

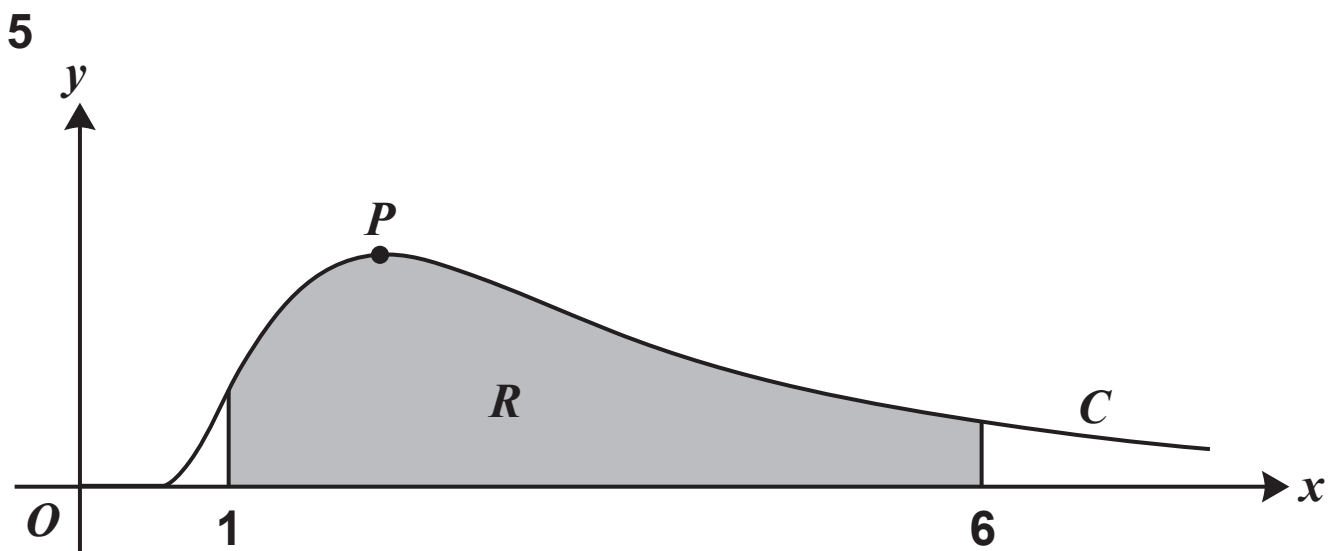
(a) Given that the curve has a point of inflection where $x = 1$, show that $k = 6$. [5]

It is also given that the curve intersects the x -axis at exactly one point.

(b) Show by calculation that the x -coordinate of this point lies between 0.5 and 1.5. [2]

(c) Use the Newton-Raphson method, with initial value $x_0 = 1$, to find the x -coordinate of the point where the curve intersects the x -axis, giving your answer correct to 5 decimal places. Show the result of each iteration to 6 decimal places. [3]

(d) By choosing suitable bounds, verify that your answer to part (c) is correct to 5 decimal places. [1]



The diagram shows the curve C with parametric equations

$$x = \frac{3}{t}, \quad y = t^3 e^{-2t}, \quad \text{where } t > 0.$$

The maximum point on C is denoted by P .

(a) Determine the exact coordinates of P . [4]

The shaded region R is enclosed by the curve, the x -axis and the lines $x = 1$ and $x = 6$.

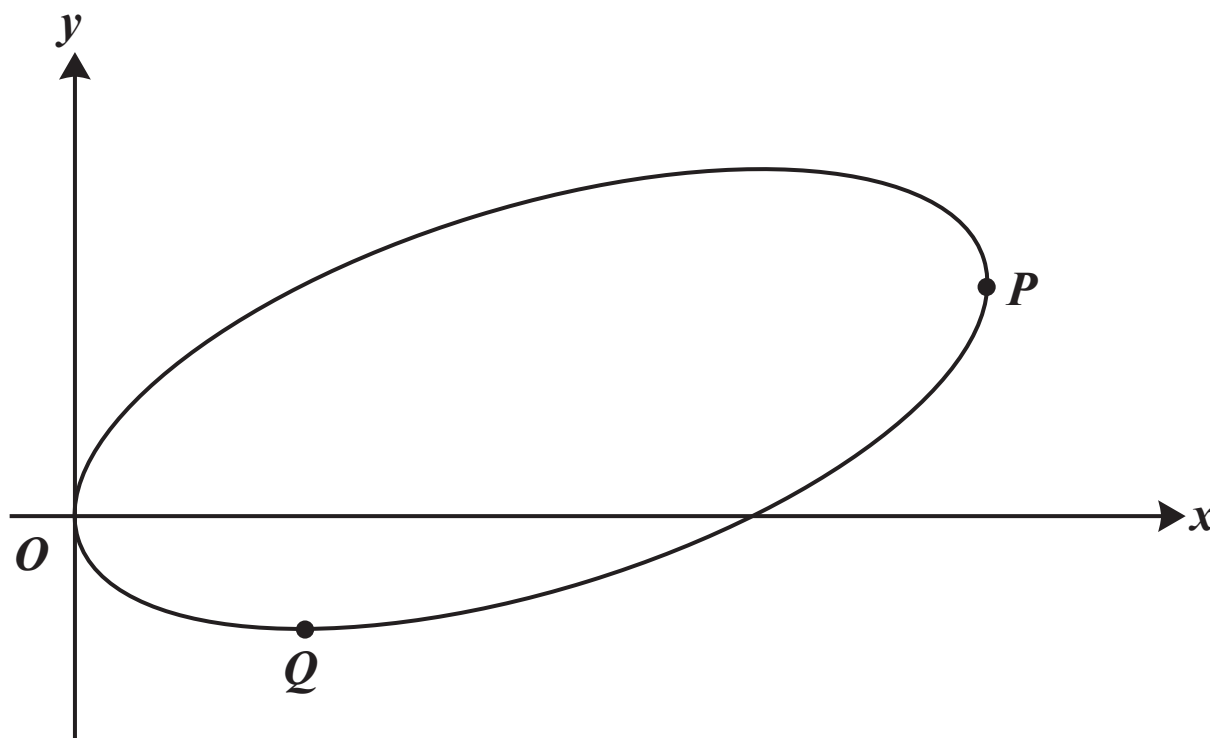
(b) Show that the area of R is given by

$$\int_a^b 3te^{-2t} dt,$$

where a and b are constants to be determined. [3]

(c) Hence determine the exact area of R . [5]

6 In this question you must show detailed reasoning.



The diagram shows the curve with equation $4xy = 2(x^2 + 4y^2) - 9x$.

(a) Show that $\frac{dy}{dx} = \frac{4x - 4y - 9}{4x - 16y}$. [3]

At the point P on the curve the tangent to the curve is parallel to the y -axis and at the point Q on the curve the tangent to the curve is parallel to the x -axis.

(b) Show that the distance PQ is $k\sqrt{5}$, where k is a rational number to be determined. [8]

SECTION B: Mechanics

Answer ALL the questions.

7 A particle P moves with constant acceleration $(-4\mathbf{i} + 2\mathbf{j})\text{ms}^{-2}$. At time $t = 0$ seconds, P is moving with velocity $(7\mathbf{i} + 6\mathbf{j})\text{ms}^{-1}$.

(a) Determine the speed of P when $t = 3$. [4]

(b) Determine the change in displacement of P between $t = 0$ and $t = 3$. [2]

- 8 A car is travelling on a straight horizontal road. The velocity of the car, $v \text{ ms}^{-1}$, at time t seconds as it travels past three points, P , Q and R , is modelled by the equation**

$$v = at^2 + bt + c,$$

where a , b and c are constants.

The car passes P at time $t = 0$ with velocity 8 ms^{-1} .

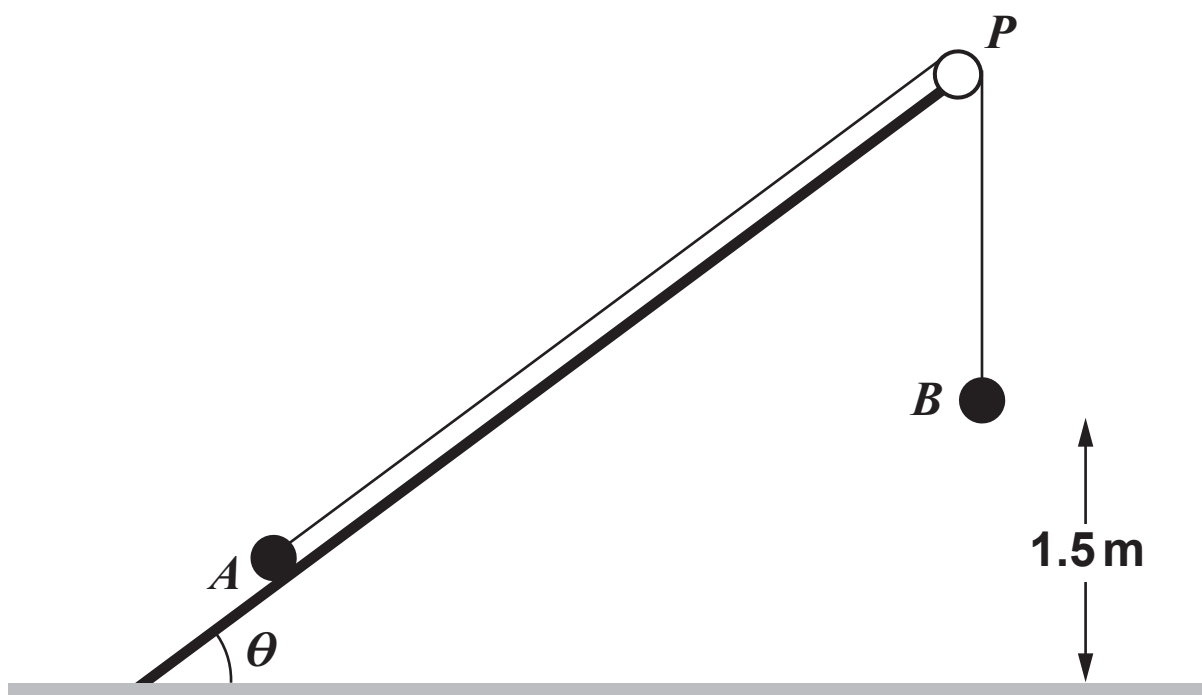
- (a) State the value of c . [1]**

The car passes Q at time $t = 5$ and at that instant its deceleration is 0.12 ms^{-2} . The car passes R at time $t = 18$ with velocity 2.96 ms^{-1} .

- (b) Determine the values of a and b . [4]**

- (c) Find, to the nearest metre, the distance between points P and R . [2]**

- 9 One end of a light inextensible string is attached to a particle A of mass 2 kg . The other end of the string is attached to a second particle B of mass 2.5 kg . Particle A is in contact with a rough plane inclined at θ to the horizontal, where $\cos \theta = \frac{4}{5}$. The string is taut and passes over a small smooth pulley P at the top of the plane. The part of the string from A to P is parallel to a line of greatest slope of the plane. Particle B hangs freely below P at a distance 1.5 m above horizontal ground, as shown in the diagram below.



The coefficient of friction between A and the plane is μ . The system is released from rest and in the subsequent motion B hits the ground before A reaches P . The speed of B at the instant that it hits the ground is 1.2 ms^{-1} .

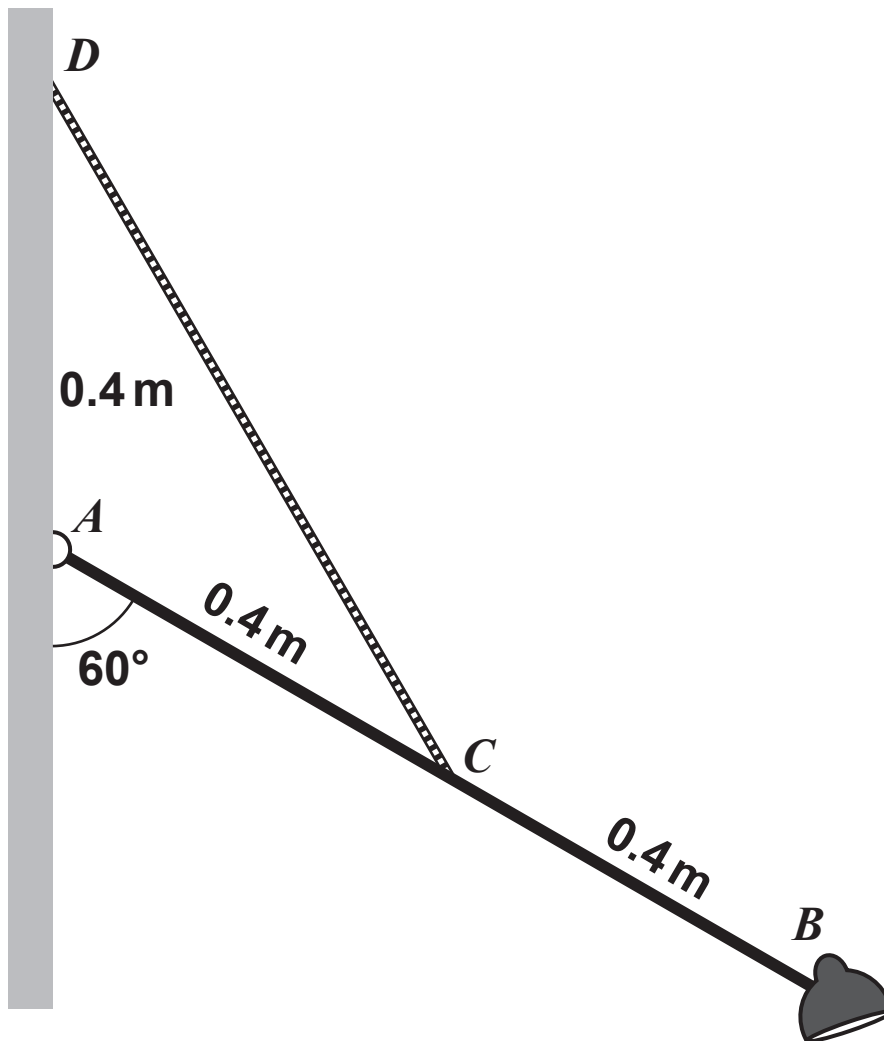
- For the motion before B hits the ground, show that the acceleration of B is 0.48 ms^{-2} . [1]
- For the motion before B hits the ground, show that the tension in the string is 23.3 N . [3]

(c) Determine the value of μ . [5]

After B hits the ground, A continues to travel up the plane before coming to instantaneous rest before it reaches P .

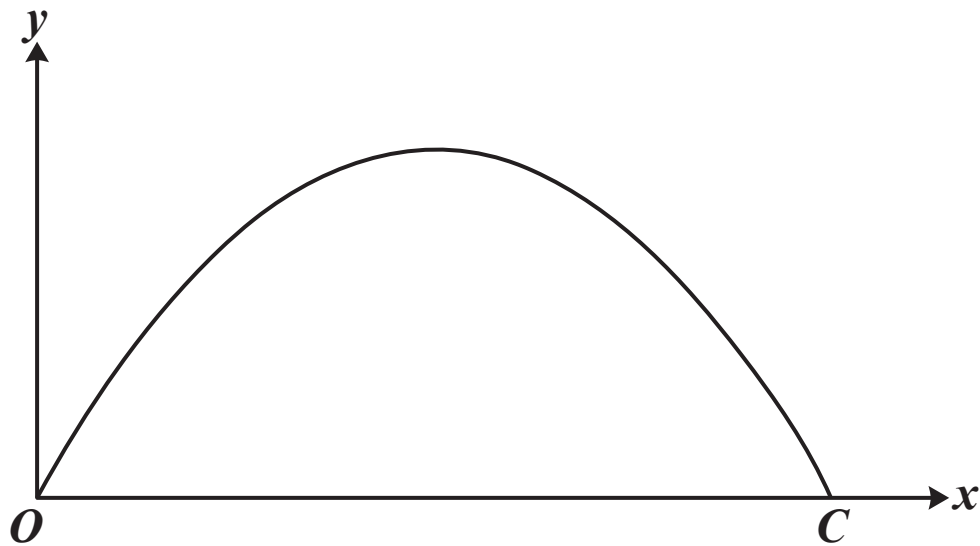
(d) Determine the distance that A travels from the instant that B hits the ground until A comes to instantaneous rest. [4]

- 10 The diagram below shows a wall-mounted light. It consists of a rod AB of mass 0.25 kg and length 0.8 m which is freely hinged to a vertical wall at A , and a lamp of mass 0.5 kg fixed at B . The system is held in equilibrium by a chain CD whose end C is attached to the midpoint of AB . The end D is fixed to the wall a distance 0.4 m vertically above A . The rod AB makes an angle of 60° with the downward vertical.



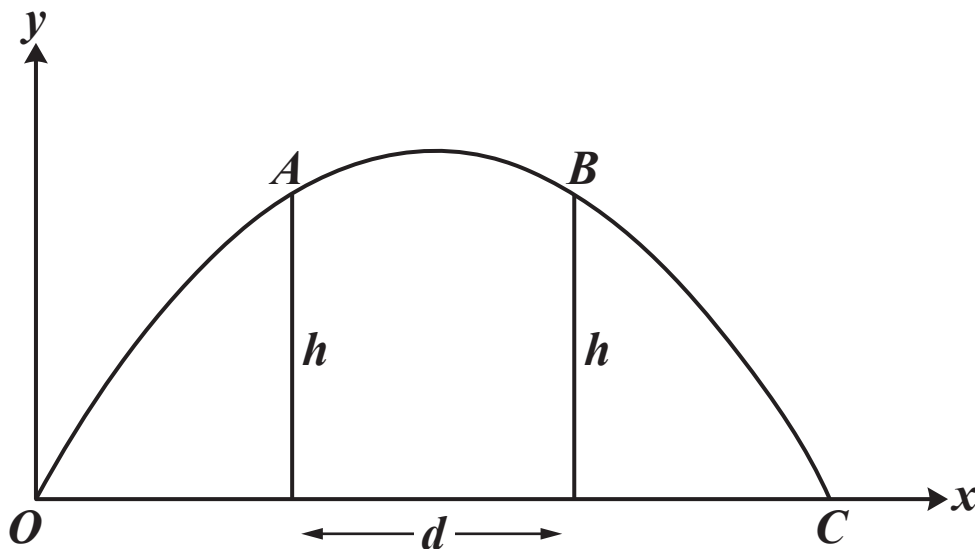
The chain is modelled as a light inextensible string, the rod is modelled as uniform and the lamp is modelled as a particle.

- (a) By taking moments about A , determine the tension in the chain. [4]
- (b) (i) Determine the magnitude of the force exerted on the rod at A . [4]
- (ii) Calculate the direction of the force exerted on the rod at A . [2]
- (c) Suggest one improvement that could be made to the model to make it more realistic. [1]



A particle P moves freely under gravity in the plane of a fixed horizontal axis Ox , which lies on horizontal ground, and a fixed vertical axis Oy . P is projected from O with a velocity whose components along Ox and Oy are U and V , respectively. P returns to the ground at a point C .

(a) Determine, in terms of U , V and g , the distance OC . [4]



P passes through two points A and B , each at a height h above the ground and a distance d apart, as shown in the diagram.

- (b) Write down the horizontal and vertical components of the velocity of P at A . [2]
- (c) Hence determine an expression for d in terms of U , V , g and h . [3]
- (d) Given that the direction of motion of P as it passes through A is inclined to the horizontal at an angle θ , where $\tan \theta = \frac{1}{2}$, determine an expression for V in terms of g , d and h . [4]

END OF QUESTION PAPER



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